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Towards more resilient integrated power grid capacity expansion: A robust optimization approach with operational flexibility

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ABSTRACT

This paper proposes a multi-period two-stage adaptive robust optimization model for long term power grid capacity expansion in an environmentally conscious way under generator output uncertainties due to natural variations and generator disruptions. In the first stage, the model plans generator addition and transmission line setup prior to, and robust to, uncertainty realization. In the second stage, it plans power generation and dispatch after, and flexible to, uncertainty realization. The model exploits the idea of strategic robustness and operational flexibility as a way to improve performance in the face of uncertainty. The robust optimization framework uses deterministic uncertainty sets, with parameters that permit control over the level of conservatism of the solution. Because the resulting two-stage model is semi-infinite and, therefore, intractable, it is reformulated as an affinely adjustable counterpart. The reformulation uses affine decision rules on second-stage variables and converts, after constraint processing, the semi-infinite model into a finite single-stage mixed integer programming model. The resulting model is tested on the IEEE 30-bus system and value gains are shown by comparing the performance of the model with a deterministic model and a stochastic programming model with varying risk aversions.

1. Introduction

Long term integrated power grid capacity expansion in an environmentally conscious way to match new load forecasts is one of the most challenging and critical problems faced by system planners in the electricity market nowadays. In recent years, the challenges of power grid capacity expansion have been compounded by the higher penetration of variable generation resources such as wind power and solar power, as well as the need for robustness against unexpected disruptions resulting from events such as terrorist attacks and natural disasters. The Fukushima-Daiichi disaster in 2011 and Hurricane Sandy in 2012 are two examples of recent natural disasters that have caused severe disruptions to power systems. This paper devises a long term plan to increase the capability of power networks in an integrated manner and gradually phase out environmentally unfriendly energy sources, in a way that is robust to natural variations and disruptions.

Today's power grids are complex systems comprising of two main components, namely generation and transmission. Generation is the production of electricity (by facilities also known as power plants) from fuel sources such as coal, natural gas, solar power, and so on. Transmission is the movement of electricity 1) at high voltages over large distances to substations and 2) at low voltages from substations to consumers. Performing generation expansion planning (GEP) and transmission expansion planning (TEP) must be performed in an integrated manner to avoid coordination issues and possible unsatisfied demands. A 2016 white paper by ABB on planning for renewable generation (see Romero and Kara, 2016) illustrates this point with the case of the West Texas power grid where, between 2007 and 2011, a significant overbuild of wind

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turbines without its matching transmission expansion occurred. The wind generation capacity was increased by around 6400 MW during that period, without the required transmission capacity addition. As a result, the additional capacity could not be brought to consumers and the additional wind generation capacity remained largely unused.

Another hindrance to demand and supply matching in power grid expansion is uncertainty. Traditionally, power grids have had to deal with demand uncertainties only. With the introduction of clean, but variable energy sources to curb greenhouse emissions, supply-side uncertainties have become a mainstay. In 2015, annual wind capacity installations reached a record 60 GW, bringing the global cumulative wind power to 432.9 GW and in 2016, the strong, albeit not record-breaking, growth of global wind energy penetration continued with 50 GW of installed capacity, bringing the global cumulative wind power to 486.8 GW (Global Wind Energy Council, 2016). Similarly, annual photovoltaic installations for solar power generation hit a record-breaking of 75 GW in 2016 (The PV Market Alliance, 2016), which is 50% higher than in 2015. With this rising prominence of renewable – and variable – energy sources, power grids have become vulnerable to supply uncertainties. If these variations are not taken into account when planning power grid expansion, severe operational malfunctions can occur. For instance, a major contributing factor to the 2000 California energy crisis was the severe drop in regional hydropower availability due to low rainfall (Sweeney, 2013). In 2008, an unexpected 1400 MW drop in wind power generation, coinciding with an unexpected load increase, forced 1100 MW load shedding in Texas (Ela and Kirby, 2008).

Because power grid infrastructures are strongly interconnected, small local deviations can cause cascading failures and result in large scale blackouts (Crucitti et al., 2004). Recent analyses of 16 years of North American disturbance data show that large blackouts are much more likely than might be expected (Chen et al., 2005). Such high-impact events, known as black swans, are in general inappropriately overlooked in planning and rationalized with the benefit of hindsight, to the detriment of system operability (Taleb, 2010). In addition, the probability of such events are difficult to estimate from historical data and available commoditized metrics (Taleb, 2007). To tackle variations, it is therefore, in a strategic sense, imperative for integrated power grid expansion plans to have robust generator and line additions, meaning that when the topology of the grid is modified, the grid must remain functional under variations in demand and supply. This functionality must be maintained even under variations where probabilities are not calculable. In an operational sense, the topology must offer the flexibility to re-route power if and when demand and supply vary. Several fields of research also provide flexibility on a strategic level through real options thinking (see Trigeorgis, 1996; Cardin, 2014; Cardin et al., 2017; Nembhard et al., 2005; Jiao, 2012; Enders et al., 2010) but these would require on-the-fly changes to generator and line additions after uncertainty observations, which may be impractical. In a 2012 report by its National Renewable Energy Laboratory on best practices in power grid expansion planning, U.S. Department of Energy identified the consideration of the inherent variability of renewable energy sources, the coordination of generation and transmission expansion planning, and the enabling of flexibility in power systems as being the keys to building greener and better grids (see Cochran et al., 2012).

To address the issues above, this paper develops a two-stage multi-period adjustable robust optimization model for integrated power grid capacity expansion planning under supply uncertainties resulting from generator disruptions and natural variations in the availability of energy sources. In the first stage, the model plans generator addition and transmission line setup prior to, and robust to, uncertainty realization. In the second stage, it plans power generation and dispatch after, and flexible to, uncertainty realizations. For simplicity, the proposed transmission expansion planning model ignores the strategic correlation effects between power generation expansion and power transmission expansion decisions, as explored in Pozo et al. (2013) and Pozo et al. (2017). The model optimizes 1) the total cost of setting up generators and lines and 2) the worst-case total power generation cost over uncertainty realizations, taking into account prescriptions for yearly environmental impact reductions. The robust optimization approach is attractive in that it does not require probability distributions and only needs moderate information about the underlying uncertainty, such as the mean and the range of the uncertain data (Bertsimas et al., 2013). It works by immunizing against all realizations of the uncertain data within a deterministic uncertainty set.

Adjustable robust counterparts, also known as robust counterparts with recourse, while offering the attractive feature of immunization against uncertainty without probability distributions together with operational flexibility through adjustable recourse decisions, are generally semi-infinite and NP-hard. For cases with relatively complete recourse, exact solution methods exist, such as the Benders decomposition approach with outer-approximated dual subproblem used by Bertsimas et al. (2013) for the security constrained unit commitment problem and the column-and-constraint generation method that generates cuts based on KKT conditions, developed by Zeng and Zhao (2013). The model developed in this paper has no clear enforceable modifications that would ensure relatively complete recourse. For such a case, an approximation is necessary. This paper applies the affine decision rule, a widely used approximation which involves converting adjustable decisions into affine combinations of uncertainty realizations. This method, developed by Ben-Tal et al. (2004), resolves intractability by yielding a linear programming model after constraint processing. To validate the decision rule, the linear programming model is applied to the IEEE 30-bus system and the results compared with that of a deterministic model and that of stochastic programming models with different risk aversions.

The main contributions of this paper can be summarized as follows:

1. A multi-period two-stage adaptive robust optimization model is formulated for long term integrated power grid capacity expansion in an environmentally conscious way. The first stage is used to plan robust generator addition and transmission line addition/expansion. The second stage plans power generation and dispatch under generator output variations, in a way that minimizes the worst-case total cost of generation cost. The uncertainty characterization of generator output variations is in the form of a deterministic uncertainty set, with parameters that permit control over the level of conservatism of the solution. The model in its current state is semi-infinite and intractable.
2. To resolve tractability issues, an affinely adjustable robust counterpart is developed for the model. The resulting constraints are then processed to convert the original model into a finite single-stage mixed integer programming model. The latter, itself a big model, is solved within an imposed time limit. Inferences about optimal value variations with changes in the level of conservatism are then used to improve the best solution obtained within the time limit.
3. The single-stage finite mixed integer model is implemented in a case study using the IEEE 30-bus system data to show the improvements it generates in power grid expansion. The first results demonstrate improvements gained compared to a deterministic model. The generator and line setup solution of the deterministic model only outperforms the robust optimization solution under the case where there are very small generator output losses. The solution of the affinely adjustable

robust counterpart is also compared with that of a stochastic model with a conditional value-at-risk objective. The conditional value-at-risk objective allows variations in the risk aversion of the decision maker as compared to the expected value objective which is better suited for the risk neutral decision maker. The results show that under extreme events, the stochastic programming model yields significantly more infeasible instances than the affinely adjustable robust counterpart. This shows greater resilience to potential black swans. Finally, the affinely adjustable robust counterpart is compared with a fully-immunized stochastic programming model whereby infeasible instances have been removed. In this case, the results show that the affinely adjustable robust counterpart clearly outperforms the stochastic programming model.

The remainder of the paper is structured as follows. Section 2 provides background on work relevant to this study. Section 3 develops the formulation of the deterministic integrated power grid expansion model and applies it on the IEEE 30-bus system. Section 4 introduces the robust counterpart of the deterministic model with its reformulation using affine decision rules. The resulting reformulated model is also applied to the IEEE 30-bus system and the results compared with those of the deterministic model and of a stochastic programming model with a conditional value-at-risk objective that allows risk aversion variations. Section 5 concludes the work and provides future work directions.

2. Literature review

This section compiles relevant literature on optimization models for power grid expansion planning, as well as robust optimization models.

2.1. Power grid expansion planning

Optimization models for power grid expansion planning are divided into 3 main types: models for GEP, models for TEP, and models for integrated planning, where both GEP and TEP are considered. GEP is the problem of determining an optimal setup and generation plan for power generators over a finite long-term planning horizon in order to meet increasing future demand for electricity. Recent papers in GEP have dealt with an array of issues, such as decentralized expansion planning (Chuang et al., 2001; Pereira and Saraiva, 2010, 2011), reliability considerations (Murugan et al., 2009), environmental impact (Karaki et al., 2002; Antunes et al., 2004; Avetisyan et al., 2006; Sirikum and Techanitisawad, 2006; Meza et al., 2007; Zangeneh et al., 2009; Tekiner et al., 2009, 2010), and cost uncertainties and risk averse objectives (Jin et al., 2011; Pisciella et al., 2016). TEP has as a core concern line addition, line switching (determining which lines to use), and power dispatch. For an introduction to the different optimization models for TEP, see Frank and Rebennack (2016). Among the issues discussed are contingency planning (Samarakoon et al., 2001; Bienstock and Mattia, 2007; Choi et al., 2007; Akbari et al., 2011), vulnerability analysis (Pinar et al., 2010), network re-design (Moulin et al., 2010), distribution expansion planning (Haffner et al., 2008a,b), price competitions (Taheri et al., 2017), payment schemes (Bravo et al., 2016), and fuel-switching decisions (Ishfaq et al., 2016). For a recent comprehensive review of the various optimization models used in GEP and TEP, see Hemmati et al. (2013).

Even in deregulated electricity markets like the ones most countries have today, it is important to integrate GEP and TEP for coordinated expansion. The choice of generating units is strongly influenced by the distance of these units from transmission substations and load centers (Sharan and Balasubramanian, 2012). In addition, when subject to disruption, a power network needs to provide a coordinated response, the effectiveness of which must be incorporated in the planning process. Several papers have integrated GEP and TEP in deterministic settings (Quelhas et al., 2007; Roh et al., 2007; Motamedi et al., 2010; Sharan and Balasubramanian, 2012; Pozo et al., 2013). Studies incorporating uncertainties (Álvarez López et al., 2007; Roh et al., 2009; Alizadeh and Jadid, 2011; Aghaei et al., 2014) do so via probabilistically distributed scenarios. The uncertainties are in demand, fuel prices, and electricity prices. Among the studies that incorporate uncertainties in the literature, the ones to incorporate supply uncertainties do so in GEP (see Jin et al., 2015) and TEP (see Jiang et al., 2016) separately. An exception is Moreira et al. (2017) where supply uncertainty is considered in terms of worst-case scenario realizations in an integrated GEP-TEP model.

2.2. Robust optimization

Robust optimization was introduced by Soyster (1973) and has gained popular outreach via the works in Ben-Tal and Nemirovski (1998, 1999, 2000), Ghaoui and Lebret (1997), Ghaoui et al. (1998), and Bertsimas and Sim (2003, 2004) which have addressed the over-conservatism of Soyster's technique. Robust optimization with adjustable decisions, usually formulated as two- or multi-stage models with recourse, are widely applied to solve models where probability distributions of uncertainty are not specified. Applications can be found in areas such as retailer-supplier contracting (Ben-Tal et al., 2005), emergency logistics planning (Ben-Tal et al., 2011), inventory control (Bertsimas and Thiele, 2006; Adida and Perakis, 2006), portfolio selection (Quaranta and Zaffaroni, 2008), and generator connection timing to power grids (Sauma et al., 2015). Since adjustable robust optimization models are generally NP-hard, approximations are used to solve them and these approximations are in the form of decision rules. A decision rule is a function that maps the observations of uncertainty data to specific decisions (Shapiro et al., 2014). Garstka and Wets (1974) provide a survey of the different types of decision rules available: zero-order, affine, safety-first and conditional-go. Affine decision rules, the most popular approximation, assume that the decision is a linear function of uncertainty realization. These decision rules are widely used in robust and stochastic optimization with recourse to make models, that are computationally cumbersome, tractable (see Kuhn et al., 2011; Chen et al., 2008; Ben-Tal and Nemirovski, 1998, 1999, 2000).

3. Deterministic model formulation

Consider a power grid with a set of nodes B and a set of edges V. The nodes are divided into a set of supply nodes S, a set of transmission nodes I, and a set of demand nodes D. The edges are divided into a set of existing lines E^L which are part of the original setup of the grid and a set of candidate lines N^L which can be used to make new connections. A multi-period model is formulated with the aim of planning four main operational decisions in each of T time periods: 1) How many of each generator type from a set K of generator types to set up at each supply node, 2) how much power to produce from each operating generator, 3) which power lines to use and to expand, and 4) how much power to send along each line. The problem formulation, together with its input data and decision variables are given as follows:

Data

r	Discount rate
c^G_{ikt}	Cost of adding a new generator of type $k \in K$ at $i \in S$ in time period t

c^L_{ijt}	Cost of adding a new line on $(i,j) \in N^L$ in time period t
c^E_{ijt}	Cost of expanding the capacity of line $(i,j) \in V$ in time period t
c^P_{ikt}	Cost per MWh of generating power from a generator type $k \in K$ at $i \in S$ in time period t
s_{ij}	Series reactance of line $(i,j) \in V$
d_{it}	Demand at node $i \in D$ in time period t (MWh)
v^U_{ikt}	Maximum output from a type $k \in K$ generator at node $i \in S$ in time period t (MWh)
n_{ik}	Number of existing generators of type $k \in K$ at node $i \in S$
γ_k	Environmental impact per MWh of power from a generator of type $k \in K$
β	Rate of reduction in total environmental impact
f^L_{ij}	Flow limit of line $(i,j) \in V$ (MWh)
f^E_{ij}	Flow added due to expansion to line $(i,j) \in V$ (MWh)

Decision Variables

x^G_{ikt}	Number of generators of type k set up at node $i \in S$ in time period t
y^L_{ijt}	Is 1 if a line is at $(i,j) \in V$ in time period t, 0 otherwise
y^E_{ijt}	Is 1 if line $(i,j) \in V$ is expanded in time period t, 0 otherwise
g_{ikt}	Power generated by generator of type $k \in K$ at node $i \in S$ in time period t
θ_{it}	Angle at node $i \in B$ in time period t
p_{ijt}	Power transmission along line $(i,j) \in V$ in time period t

Model

$$\min \sum_{t=1}^{T} \frac{1}{(1+r)^t} \left(\sum_{i \in S} \sum_{k \in K} c^G_{ikt} x^G_{ikt} + \sum_{(i,j) \in N^L} c^L_{ijt} y^L_{ijt} + \sum_{(i,j) \in V} c^E_{ijt} y^E_{ijt} + \sum_{i \in S} \sum_{k \in K} c^P_{ikt} g_{ikt} \right)$$

$$\text{s.t.} \quad s_{ij}(\theta_{it} - \theta_{jt}) = p_{ijt} \quad \forall (i,j) \in V, t \in \{1,\ldots,T\} \tag{1}$$

$$\sum_{(i,j) \in \sigma_i^-} p_{ijt} - \sum_{(j,i) \in \sigma_i^+} p_{jit} = \sum_{k \in K} g_{ikt} \quad \forall i \in S, t \in \{1,\ldots,T\} \tag{2}$$

$$\sum_{(j,i) \in \sigma_i^+} p_{jit} - \sum_{(i,j) \in \sigma_i^-} p_{ijt} = d_{it} \quad \forall i \in D, t \in \{1,\ldots,T\} \tag{3}$$

$$\sum_{(j,i) \in \sigma_i^+} p_{jit} - \sum_{(i,j) \in \sigma_i^-} p_{ijt} = 0 \quad \forall i \in I, t \in \{1,\ldots,T\} \tag{4}$$

$$g_{ikt} \le v^U_{ikt} \left(n_{ik} + \sum_{\tau=1}^{t} x^G_{ik\tau} \right) \quad \forall i \in S, k \in K, t \in \{1,\ldots,T\} \tag{5}$$

$$\sum_{i \in S} \sum_{k \in K} \gamma_k g_{ikt} \le (1-\beta)^t \sum_{i \in S} \sum_{k \in K} \gamma_k g_{ik0} \quad \forall t \in \{1,\ldots,T\} \tag{6}$$

$$|p_{ijt}| \le f^L_{ij} \sum_{\tau=1}^{t} y^L_{ij\tau} + f^E_{ij} \sum_{\tau=1}^{t} y^E_{ij\tau} \quad \forall (i,j) \in N^L, t \in \{1,\ldots,T\} \tag{7}$$

$$|p_{ijt}| \le f^L_{ij} + f^E_{ij} \sum_{\tau=1}^{t} y^E_{ij\tau} \quad \forall (i,j) \in E^L, t \in \{1,\ldots,T\} \tag{8}$$

$$y^E_{ijt} \le y^L_{ijt} \quad \forall (i,j) \in N^L, t \in \{1,\ldots,T\} \tag{9}$$

$$\sum_{t=1}^{T} y^L_{ijt} \le 1 \quad \forall (i,j) \in N^L \tag{10}$$

$$\sum_{t=1}^{T} y^E_{ijt} \le 1 \quad \forall (i,j) \in V \tag{11}$$

All $y^L_{ijt}, y^E_{ijt} \in \{0,1\}$, all $x^G_{ikt} \in \mathbb{Z}_+$, all $p_{ijt}, \theta_{it} \in \mathbb{R}$, all $g_{ikt} \in \mathbb{R}_+$.

Note that in practice, some nodes can be both supply and demand nodes. The model presented here does not restrict this, in the sense that these nodes can be listed both in set S and set D. The separation of grid nodes into demand, supply, and transmission nodes is to keep the model well-structured and easy to understand. The objective of the model is to minimize the total discounted cost, over the planning horizon, of setting up generators and lines and of power generation. Constraint (1) is the DC flow constraint. Constraints (2), (3), and (4) are flow balance constraints, where σ_i^+ is the set of lines entering node $i \in B$ and σ_i^- is the set of lines leaving node $i \in B$. Flow balance constraints ensure that: 1) the net outflow at a supply node is the amount of power generated by operating generators at that node, 2) the net inflow at a demand point is equal to its demand, and 3) the power inflow at a transmission node equals the power outflow. Constraint (5) limits the generation capacity from both new and existing generators. Note that it is unnecessary, in this model, to separately track whether generation comes from a new or an existing generator, given that the generation cost is the same for a type of generator, whether it is new or existing. The power capacity in our model is expressed in MWh. This allows the consideration of power generated over a whole time period, instead of in a snapshot of the time period (which would be the case if the capacity ratings of generators in MW were to be used). Such usage of units is employed such works as Sirikum and Techanitisawad (2006), and Caunhye and Cardin (2017). Constraints (6) enforce environmental impact reduction in every period. Note that $\mathbf{g}_0$ is the vector of generation amounts in the initial power grid (prior to expansion). We assume that the environmental impact varies linearly with the generation amount for every generator type. This is a necessary simplification to maintain computational tractability in the model. Constraints (7), and (8) are line capacity constraints. Constraint (9) ensures that a line can only be expanded if it has been set up. Constraints (11) and (10) make sure that a line cannot be added or expanded more than once. Note that the generator addition variables maintain a discrete nature here. In the future, as the deployment of generators become more and more modular, it is likely that continuous variables could be used as close approximations of generator deployment variables.

3.1. Case study of the IEEE 30-bus system - deterministic

To demonstrate how the deterministic model works, it is applied to the standard IEEE 30-bus system shown in Fig. 1 with a planning horizon of 3 years. The system line and bus data are obtained from Lee et al. (1985). The load on every node for the system in initial state is shown in Table 1. The load growth rate is assumed to be 3%, based on the 2015 EIA report on the annual energy outlook (see U.S. Energy Information Administration, 2015).

Line data are shown in Table 2. Expansion and setup costs are assumed to be proportional to reactance, with the same proportionality as given in Alguacil et al. (2003). Line expansion is equivalent to the addition of another line of the same type. In the initial state, the power grid only has a coal generator at each of nodes 1, 2, 5, 8, 11, 13 with generator ratings of 150 MW, 60 MW, 40 MW, 25 MW, 20 MW, and 30 MW respectively. The planner intends to set up

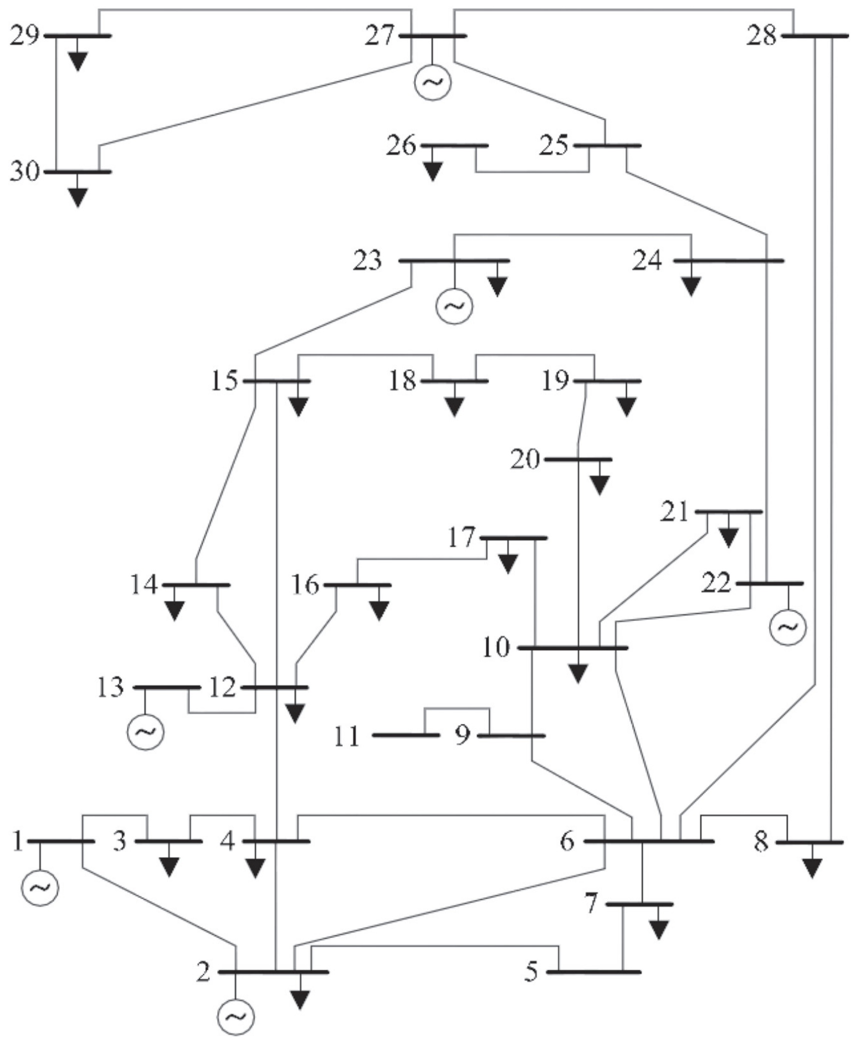

Fig. 1. IEEE 30-bus system single-line diagram (Abido, 2002).

generators from 4 types, advanced combustion turbine (ACT), conventional combustion turbine (CCT), Photovoltaic model (PVM), and wind farm (WF), with the characteristics of each shown in Table 3. The characteristics are obtained from the 2013 U.S. Energy Information Administration Capital Cost Estimates report (see U.S. Energy Information Administration, 2013). Coal generators have a fuel cost of \$3337 per MMBtu of heat, variable O&M cost of \$4.47 per MWh, and a total emissions amount of 1814 lb per MWh. They are omitted from the table because the planner does not intend to set up new coal generators.

The setup cost of a generator is the sum of its capital cost and its fixed O&M cost. The generation cost is the variable O&M cost and the environmental impact is measured in the total volume of emissions generated. The mixed integer programming model is coded in C++ and Concert Technology is used to solve it with IBM-ILOG CPLEX 12.6 on a 2.4 GHz core i7-4500U processor with 12 GB of RAM. Suppose the discount rate is 0.08 and the planner has a mandate to reduce the total emissions by a minimum 30% every year for the next 3 years. It takes 1.78 s to solve the model. Table 4 shows the generation expansion plan, which consists of generator addition and power generation. A major observation from the plan is the gradual replacement of the environmentally unfriendly coal with clean zero-emission sources (wind power and solar power) of energy. The percentage of demand supplied by coal decreases from 100% in the initial state to 65.7% in the first year, then further to 33.5% in the second year, and finally to 29.0% in the third year. The optimal total cost of installation and generation is \$4.97 $\times$ 10^8. In terms of transmission expansion, a new line is added from node 15 to 27 in the first time period and no lines are expanded.

Table 1
Initial state load data.

Node	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Load (MW)	0.0	21.7	2.4	7.6	94.2	0.0	22.8	30.0	0.0	5.8	0.0	11.2	0.0	6.2	8.2
Node	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Load (MW)	3.5	9.0	3.2	9.5	2.2	17.5	0.0	3.2	8.7	0.0	3.5	0.0	0.0	2.4	10.6

Table 2
Line data.

Existing line	1–2	1–3	2–4	3–4	2–5	2–6	4–6	5–7	6–7	6–8	6–9	6–10
Rating (MW)	130	130	65	130	130	65	90	70	130	32	65	32
Reactance (p.u.)	0.06	0.19	0.17	0.04	0.20	0.18	0.04	0.12	0.08	0.04	0.21	0.56
Expansion cost (million $)	5.8	18.5	17.4	3.8	19.8	17.6	4.1	11.6	8.2	4.2	20.8	55.6
Existing line	9–11	9–10	4–12	12–13	12–14	12–15	12–16	14–15	16–17	15–18	18–19	19–20
Rating (MW)	65	65	65	65	32	32	32	16	16	16	16	32
Reactance (p.u.)	0.21	0.11	0.26	0.14	0.26	0.13	0.20	0.20	0.19	0.22	0.13	0.07
Expansion cost (million $)	20.8	11.0	25.6	14.0	25.6	13.0	19.9	20.0	19.3	21.9	12.9	6.8
Existing line	10–20	10–17	10–21	10–22	21–22	15–23	22–24	23–24	24–25	25–26	25–27	28–27
Rating (MW)	32	32	32	32	32	16	16	16	16	16	16	65
Reactance (p.u.)	0.21	0.08	0.07	0.15	0.02	0.20	0.18	0.27	0.33	0.38	0.21	0.40
Expansion cost (million $)	20.9	8.5	7.5	15.0	2.4	20.2	17.9	27.0	32.9	38.0	20.9	39.6
Existing line	27–29	27–30	29–30	8–28	6–28							
Rating (MW)	16	16	16	32	32							
Reactance (p.u.)	0.42	0.60	0.45	0.20	0.06							
Expansion cost (million $)	41.5	60.3	45.3	20.0	6.0							
Candidate line	15–27	7–8										
Rating (MW)	30	30										
Reactance (p.u.)	0.14	0.11										
Setup cost (million $)	14.0	11.0										

Table 3
Data for new generators.

Generator type	Rating (MW)	Capital cost ($)	Fixed O&M ($/year)	Variable O&M ($/MWh)	Heat rate (MMBtu/ MWh)	Emissions		
						SO_2(lb/ MMBtu)	NO_x(lb/ MMBtu)	CO_2(lb/ MMBtu)
ACT	210	141,960,000	1,478,400	10.37	9.75	0.001	0.030	117
CCT	85	82,705,000	623,900	15.45	10.85	0.001	0.030	117
PVM	20	83,660,000	555,000	0	N/A	0	0	0
WF	100	221,300,000	3,955,000	0	N/A	0	0	0

4. Robust counterpart

The model built in the previous section assumes all parameter values to be known with certainty, which in reality is not the case. For example, the maximum output of a photovoltaic module depends on available sunlight, which varies daily and seasonally. Typically, the main uncertainties in power grid expansion planning are in demand and supply. In a robust optimization setting, these uncertainties are characterized by deterministic uncertainty sets, unlike the stochastic optimization case where probability distributions are involved. Here, only

Table 4
Optimal generation expansion plan.

Period	Supply nodes					
	Node 1	Node 2	Node 5	Node 8	Node 11	Node 13
1	– Decrease coal power (from initial state value of 1.19×10^6 MWh) to 3.49×10^5 MWh. – Set up a wind farm and generate 8.76×10^5 MWh from wind power	– No action (maintain coal power at its initial state value of 5.26×10^5 MWh).	– No action (maintain coal power at its initial state value of 3.50×10^5 MWh).	– Decrease coal power (from initial state value of 2.01×10^5 MWh) to 1.96×10^5 MWh.	– No action (maintain coal power at its initial state value of 1.75×10^5 MWh)	– Increase coal power (from initial state value of 4.06×10^4 MWh) to 8.41×10^4 MWh.
2	– Stop coal power.	– Stop coal power. – Set up a wind farm and generate 8.76×10^5 MWh from wind power.	– Decrease coal power to 3.24×10^5 MWh.	– Decrease coal power to 1.88×10^5 MWh.	– No action.	– Increase coal power to 1.94×10^5 MWh.
3	– Increase coal power to 5.11×10^5 MWh.	– No action.	– Stop coal power.	– Decrease coal power to 9.85×10^4 MWh.	– No action.	– Decrease coal power to 8.82×10^3 MWh. – Set up a photovoltaic module and generate 1.75×10^5 MWh from solar power.

supply uncertainties are considered. While demand uncertainties do obviously exist as well, they are omitted with the thought of building a power grid that can handle peak demands on all demand points. Supply uncertainties, which affect generator outputs, come from sources such as natural variations in resource availability and generator efficiency and breakdowns due to catastrophic events or malfunctions. A resilient power grid is one that maintains full load satisfaction under these variations.

The uncertainty set for a time period t is characterized as the polyhedron:

$$\mathcal{U}_t\left(\boldsymbol{v}_t^{\boldsymbol{L}}, \boldsymbol{v}_t^{\boldsymbol{U}}, \Gamma_t\right) = \left\{\boldsymbol{v}_t \in \mathbb{R}_+^{|S|\times|K|} : v_{ikt}^L \leq v_{ikt} \leq v_{ikt}^U, \forall i \in S, k \in K, \Gamma_t \sum_{i\in S}\sum_{k\in K} v_{ikt}^L \leq \sum_{i\in S}\sum_{k\in K} v_{ikt}\right\}$$

The output of a generator of type k at supply node i in time period t, v_{ikt}, has a range of values $\left[v_{ikt}^L, v_{ikt}^U\right]$. The upper bound, v_{ikt}^U, is its maximum output possible. The lower bound, v_{ikt}^L, accounts for natural variations, efficiency losses, downtimes due to maintenance, and complete breakdowns during the time period. The term v_{ikt} represents the total MWh output of a generator of type k at supply node i over the entirety of the time period t (usually a year) and the range $\left[v_{ikt}^L, v_{ikt}^U\right]$ can be used to represent situations of complete generator breakdowns even when $v_{ikt}^L > 0$. For example, a situation $v_{ikt}^L = 0.5v_{ikt}^U$ could represent a case where generators of type k at node i worked at full capacity for half of year t, before disaster struck and forced a complete shutdown of these generators. As such, uncertainty characterization using ranges of values can represent a wide range of scenarios. Situations where complete losses occur generally result from high-impact low-probability events and are often inappropriately overlooked in planning and rationalized with the benefit of hindsight (see the black swan theory Musgrave and Taleb, 2009). In this paper, irrespective of the probability of occurrence of such events, the model seeks to immunize the grid against them. It is important to note that, for simplicity, the uncertainty set does not cover correlations and non-linearities. Indeed, disaster impacts are potentially non-linear, with cascading effects and multiple nodes being taken out altogether. Capturing such non-linearities and correlations adequately is challenging and, even when possible, could render the solution challenging to attain. The parameter $1 \leq \Gamma_t \leq \frac{\sum_{i\in S}\sum_{k\in K} v_{ikt}^U}{\sum_{i\in S}\sum_{k\in K} v_{ikt}^L}$ is the budget of uncertainty and is used to provide control over the level of robustness and prevent over-conservatism of solutions. Indeed, a trade-off exists between performance and protection against uncertainty. Given that recourse decisions made in a time period, t, depend on $\boldsymbol{v}^t$ $(= \{\boldsymbol{v}_1, \ldots, \boldsymbol{v}_t\})$, the history of generator output realizations from the first time period up to t, the formulation of the robust counterpart is:

$$\min \sum_{t=1}^{T} \frac{1}{(1+r)^t}\left(\sum_{i\in S}\sum_{k\in K} c_{ikt}^G x_{ikt}^G + \sum_{(i,j)\in N^L} c_{ijt}^L y_{ijt}^L + \sum_{(i,j)\in V} c_{ijt}^E y_{ijt}^E\right) + \sum_{t=1}^{T} \max_{\boldsymbol{v}^t \in \prod_{\tau=1}^t \mathcal{U}_\tau} \sum_{i\in S}\sum_{k\in K} \frac{c_{ikt}^P}{(1+r)^t} g_{ikt}(\boldsymbol{v^t})$$

$$\text{s.t.}\quad \forall \boldsymbol{v}^t \in \prod_{\tau=1}^{t} \mathcal{U}_\tau, t \in \{1,\ldots,T\}, \exists p_{ijt}(.) \in \mathbb{R}, \forall (i,j) \in V, \theta_{it}(.) \in \mathbb{R}, \forall i \in B, g_{ikt}(.) \in \mathbb{R}_+, \forall i \in S, k \in K$$

$$\text{s.t.}\quad s_{ij}\left(\theta_{it}(\boldsymbol{v}^t) - \theta_{jt}(\boldsymbol{v}^t)\right) = p_{ijt}(\boldsymbol{v}^t) \quad \forall (i,j) \in V, t \in \{1,\ldots,T\}$$

$$\sum_{(i,j)\in\sigma_i^-} p_{ijt}(\boldsymbol{v}^t) - \sum_{(j,i)\in\sigma_i^+} p_{jit}(\boldsymbol{v}^t) = \sum_{k\in K} g_{ikt}(\boldsymbol{v}^t) \quad \forall i \in S, t \in \{1,\ldots,T\}$$

$$\sum_{(j,i)\in\sigma_i^+} p_{jit}(\boldsymbol{v}^t) - \sum_{(i,j)\in\sigma_i^-} p_{ijt}(\boldsymbol{v}^t) = d_{it} \quad \forall i \in D, t \in \{1,\ldots,T\}$$

$$\sum_{(j,i)\in\sigma_i^+} p_{jit}(\boldsymbol{v}^t) - \sum_{(i,j)\in\sigma_i^-} p_{ijt}(\boldsymbol{v}^t) = 0 \quad \forall i \in I, t \in \{1,\ldots,T\}$$

$$g_{ikt}(\boldsymbol{v}^t) \leq v_{ikt}\left(n_{ik} + \sum_{\tau=1}^{t} x_{ik\tau}^G\right) \quad \forall i \in S, k \in K, t \in \{1,\ldots,T\}$$

$$\sum_{i\in S}\sum_{k\in K} \gamma_k g_{ikt}(\boldsymbol{v}^t) \leq (1-\beta)^t \sum_{i\in S}\sum_{k\in K} \gamma_k g_{ik0} \quad \forall t \in \{1,\ldots,T\}$$

$$\left|p_{ijt}(\boldsymbol{v}^t)\right| \leq f_{ij}^L \sum_{\tau=1}^{t} y_{ij\tau}^L + f_{ij}^E \sum_{\tau=1}^{t} y_{ij\tau}^E \quad \forall (i,j) \in N^L, t \in \{1,\ldots,T\}$$

$$\left|p_{ijt}(\boldsymbol{v}^t)\right| \leq f_{ij}^L + f_{ij}^E \sum_{\tau=1}^{t} y_{ij\tau}^E \quad \forall (i,j) \in E^L, t \in \{1,\ldots,T\}$$

$$y_{ijt}^E \leq y_{ijt}^L \quad \forall (i,j) \in N^L, t \in \{1,\ldots,T\}$$

$$\sum_{t=1}^{T} y_{ijt}^L \leq 1 \quad \forall (i,j) \in N^L$$

$$\sum_{t=1}^{T} y_{ijt}^E \leq 1 \quad \forall (i,j) \in V$$

$$\text{All}\quad y_{ijt}^L, y_{ijt}^E \in \{0,1\},\ \text{all}\ x_{ikt}^G \in \mathbb{Z}_+.$$

4.1. Affinely adjustable robust counterpart

The first constraint enforces the feasibility of the second-stage model for every possible uncertainty realization. It is referred to as an immunization condition. The robust counterpart is a semi-infinite problem in that it has an infinite number of constraints due to the immunization condition and a finite number of decision variables. Bertsimas et al. (2013) use a Benders decomposition approach to solve a semi-infinite robust counterpart for the security constrained unit commitment problem. The approach relies on converting the two-stage model into a min-max-min problem and then dualizing the inner min problem to obtain a maximization dual subproblem from which cuts are generated. Because the dual subproblem is nonlinear, an outer-approximation approach is used to solve it and obtain inexact cuts. The convergence of the Benders decomposition algorithm with inexact cuts is not guaranteed and it requires the model to have relatively complete recourse (Zakeri et al., 2000). In our case, it is easy to see that the model in its current state does not have relatively complete recourse. Furthermore, it is not clear whether such a condition is enforceable by modifying the model. The other available method to solve semi-infinite robust two-stage optimization models exactly, the column-and-constraints generation method (Zeng and Zhao, 2013), also requires relatively complete recourse.

To our knowledge, no exact solution methods are available to solve our model. To satisfy the immunization condition and yet maintain finiteness and tractability in the robust counterpart, second-stage decisions are approximated as affine functions of uncertainty. This method, which results in what is known as an affinely adjustable robust counterpart (AARC), was developed by Ben-Tal et al. (2004) and is a popular way to solve two-stage robust optimization models. AARC has been used in interesting application areas such as designing supplier-retailer flexible commitment contracts (Ben-Tal et al., 2005) and distribution systems with transshipment (Chou et al., 2006). The AARC is appealing because it is a linear programming or second order conic programming problem that can be solved efficiently with generic solvers such as CPLEX.

In the AARC, the vector of second-stage decision variables $\boldsymbol{A}_t(\boldsymbol{v}^t) \in \mathbb{R}^{\mathcal{I}}$, where $\mathcal{I}$ is the number of second-stage decision variables (e.g. an element of the vector would be g_{ikt}) is replaced using the affine relationship $\boldsymbol{A}_t(\boldsymbol{v}^t) = \boldsymbol{A}_t^0 + \sum_{i\in S}\sum_{k\in K}\sum_{\tau=1}^{t} \boldsymbol{A}_t^{ik\tau} v_{ik\tau}$. where $\boldsymbol{A}_t^0, \boldsymbol{A}_t^{ik\tau} \in \mathbb{R}^{\mathcal{I}}$. This results in the following model:

$$\min \sum_{t=1}^{T} \frac{1}{(1+r)^t}\left(\sum_{i\in S}\sum_{k\in K} c_{ikt}^G x_{ikt}^G + \sum_{(i,j)\in N^L} c_{ijt}^L y_{ijt}^L + \sum_{(i,j)\in V} c_{ijt}^E y_{ijt}^E\right) + \sum_{t=1}^{T} Q_t$$

$$\text{s.t.} \quad y_{ijt}^E \le y_{ijt}^L \quad \forall (i,j) \in N^L, t \in \{1,\ldots,T\}$$

$$\sum_{t=1}^{T} y_{ijt}^L \le 1 \quad \forall (i,j) \in N^L$$

$$\sum_{t=1}^{T} y_{ijt}^E \le 1 \quad \forall (i,j) \in V$$

$$\left.\begin{array}{l}
\sum_{i'\in S}\sum_{k'\in K}\sum_{\tau=1}^{t}\left(s_{ij}\theta_{it}^{i'k'\tau} - s_{ij}\theta_{jt}^{i'k'\tau} - p_{ijt}^{i'k'\tau}\right) v_{i'k'\tau} = p_{ijt}^0 - s_{ij}\left(\theta_{it}^0 - \theta_{jt}^0\right) \quad \forall (i,j) \in V \\
\sum_{i'\in S}\sum_{k'\in K}\sum_{\tau=1}^{t}\left(\sum_{(i,j)\in\sigma_i^-} p_{ijt}^{i'k'\tau} - \sum_{(j,i)\in\sigma_i^+} p_{jit}^{i'k'\tau} - \sum_{k\in K} g_{ikt}^{i'k'\tau}\right) v_{i'k'\tau} \\
= \sum_{k\in K} g_{ikt}^0 - \sum_{(i,j)\in\sigma_i^-} p_{ijt}^0 + \sum_{(j,i)\in\sigma_i^+} p_{jit}^0 \quad \forall i \in S \\
\sum_{i'\in S}\sum_{k'\in K}\sum_{\tau=1}^{t}\left(\sum_{(j,i)\in\sigma_i^+} p_{jit}^{i'k'\tau} - \sum_{(i,j)\in\sigma_i^-} p_{ijt}^{i'k'\tau}\right) v_{i'k'\tau} = \sum_{(i,j)\in\sigma_i^-} p_{ijt}^0 - \sum_{(j,i)\in\sigma_i^+} p_{jit}^0 + d_{it} \quad \forall i \in D \\
\sum_{i'\in S}\sum_{k'\in K}\sum_{\tau=1}^{t}\left(\sum_{(j,i)\in\sigma_i^+} p_{jit}^{i'k'\tau} - \sum_{(i,j)\in\sigma_i^-} p_{ijt}^{i'k'\tau}\right) v_{i'k'\tau} = \sum_{(i,j)\in\sigma_i^-} p_{ijt}^0 - \sum_{(j,i)\in\sigma_i^+} p_{jit}^0 \quad \forall i \in I \\
\sum_{i'\in S}\sum_{k'\in K}\sum_{\tau=1}^{t} g_{ikt}^{i'k'\tau} v_{i'k'\tau} - \left(n_{ik} + \sum_{\tau=1}^{t} x_{ik\tau}^G\right) v_{ikt} \le -g_{ikt}^0 \quad \forall i \in S, k \in K \\
\sum_{i\in S}\sum_{k\in K}\sum_{i'\in S}\sum_{k'\in K}\sum_{\tau=1}^{t} \gamma_k g_{ikt}^{i'k'\tau} v_{i'k'\tau} \le (1-\beta)^t \sum_{i\in S}\sum_{k\in K} \gamma_k g_{ik0} - \sum_{i\in S}\sum_{k\in K} \gamma_k g_{ikt}^0 \\
\left|\sum_{i'\in S}\sum_{k'\in K}\sum_{\tau=1}^{t} p_{ijt}^{i'k'\tau} v_{i'k'\tau} + p_{ijt}^0\right| \le f_{ij}^L \sum_{\tau=1}^{t} y_{ij\tau}^L + f_{ij}^E \sum_{\tau=1}^{t} y_{ij\tau}^E \quad \forall (i,j) \in N^L \\
\left|\sum_{i'\in S}\sum_{k'\in K}\sum_{\tau=1}^{t} p_{ijt}^{i'k'\tau} v_{i'k'\tau} + p_{ijt}^0\right| \le f_{ij}^L + f_{ij}^E \sum_{\tau=1}^{t} y_{ij\tau}^E \quad \forall (i,j) \in E^L \\
\sum_{i\in S}\sum_{k\in K}\sum_{i'\in S}\sum_{k'\in K}\sum_{\tau=1}^{t} \frac{c_{ikt}^P}{(1+r)^t} g_{ikt}^{i'k'\tau} v_{i'k'\tau} \le Q_t - \sum_{i\in S}\sum_{k\in K} \frac{c_{ikt}^P}{(1+r)^t} g_{ikt}^0 \\
\sum_{i'\in S}\sum_{k'\in K}\sum_{\tau=1}^{t} g_{ikt}^{i'k'\tau} v_{i'k'\tau} \ge -g_{ikt}^0 \quad \forall i \in S, k \in K
\end{array}\right\} \begin{array}{l}\forall \boldsymbol{v}^t \in \prod_{\tau=1}^{t} \mathcal{U}_\tau, \\ t \in \{1,\ldots,T\}\end{array}$$

All $y_{ijt}^L, y_{ijt}^E \in \{0,1\}$, all $x_{ikt}^G \in \mathbb{Z}_+$, all remaining variables $\in \mathbb{R}$.

The AARC in the current form is still semi-infinite and intractable. However, with some constraint processing, it can be turned into a finite mixed integer programming model. Below is an illustration of how to process the different types of constraints in the AARC with examples:

Equality constraints

Working with the constraint $\sum_{i'\in S}\sum_{k'\in K}\sum_{\tau=1}^{t}\left(s_{ij}\theta_{it}^{i'k'\tau}-s_{ij}\theta_{jt}^{i'k'\tau}-p_{ijt}^{i'k'\tau}\right)v_{i'k'\tau}=p_{ijt}^{0}-s_{ij}\left(\theta_{it}^{0}-\theta_{jt}^{0}\right)$ for a particular $(i,j)\in V$ as example: The constraint must be valid $\forall \boldsymbol{v}^{t}\in\prod_{\tau=1}^{t}\mathcal{U}_{\tau}, t\in\{1,\ldots,T\}$. This happens when

$$p_{ijt}^{0}=s_{ij}\left(\theta_{it}^{0}-\theta_{jt}^{0}\right)\quad \forall t\in\{1,\ldots,T\}$$

and

$$p_{ijt}^{i'k'\tau}=s_{ij}\left(\theta_{it}^{i'k'\tau}-\theta_{jt}^{i'k'\tau}\right)\quad \forall\tau\in\{1,\ldots,t\}, t\in\{1,\ldots,T\}, i'\in S, k'\in K.$$

Therefore, the infinite set of constraints is converted to two finite sets of equality constraints.

≤ constraints

Taking the set of constraints (for an $i\in S$ and $k\in K$) $\sum_{i'\in S}\sum_{k'\in K}\sum_{\tau=1}^{t}g_{ikt}^{i'k'\tau}v_{i'k'\tau}-\left(n_{ik}+\sum_{\tau=1}^{t}x_{ik\tau}^{G}\right)v_{ikt}\le -g_{ikt}^{0}\ \forall\boldsymbol{v}^{t}\in\prod_{\tau=1}^{t}\mathcal{U}_{\tau}, t\in\{1,\ldots,T\}$ as an example, the constraints are valid if and only if

$$\max_{\boldsymbol{v}^{t}\in\prod_{\tau=1}^{t}\mathcal{U}_{\tau}}\sum_{i'\in S}\sum_{k'\in K}\sum_{\tau=1}^{t}g_{ikt}^{i'k'\tau}v_{i'k'\tau}-\left(n_{ik}+\sum_{\tau=1}^{t}x_{ik\tau}^{G}\right)v_{ikt}\le -g_{ikt}^{0}\quad\forall t\in\{1,\ldots,T\}.$$

Because of strong duality, this set of (nonlinear) inequalities can be replaced by the system of linear inequalities

$$\sum_{i'\in S}\sum_{k'\in K}\sum_{\tau=1}^{t}\pi_{i'k'\tau}^{B}v_{i'k'\tau}^{U}-\sum_{i'\in S}\sum_{k'\in K}\sum_{\tau=1}^{t}\pi_{i'k'\tau}^{C}v_{i'k'\tau}^{L}-\sum_{\tau=1}^{t}\pi_{\tau}^{A}\Gamma_{\tau}\sum_{i'\in S}\sum_{k'\in K}v_{i'k'\tau}^{L}\le -g_{ikt}^{0}\quad t\in\{1,\ldots,T\}$$
$$\pi_{i'k'\tau}^{B}-\pi_{i'k'\tau}^{C}-\pi_{\tau}^{A}\ge g_{ikt}^{i'k'\tau}-\delta_{ikt}\left(n_{ik}+\sum_{\tau'=1}^{t}x_{ik\tau'}^{G}\right)\quad\forall\tau\in\{1,\ldots,t\}, t\in\{1,\ldots,T\}, i'\in S, k'\in K,$$

where $\boldsymbol{\pi^{A}}$, $\boldsymbol{\pi^{B}}$, and $\boldsymbol{\pi^{C}}$ are dual variables and $\delta_{ikt}=1$ if $i'=i$, $k'=k$, and $\tau=t$, and $\delta_{ikt}=0$ otherwise. Notice that all ≥ constraints can be converted to ≤ constraints. With the reformulation of all equality and inequality constraints, our AARC becomes a finite mixed integer programming model.

4.2. Case study of the IEEE 30-bus system - robust

The deterministic solution found in Subsection 2.1 works under the assumption that all parameter values are known. One of the main difficulties in integrating renewable energy sources in power grids is the uncertainty in supply capacities resulting from day-to-day or seasonal variations in the availability of these energy sources. For instance, the maximum output of photovoltaic plants can drop by up to 50% during winter (see Velamuri and Sreejith, 2015 for a study of seasonal solar energy variations). Similarly, the average power output from wind turbine generators is around half of its maximum output (see Moura and De Almeida, 2010 and Piwko et al., 2006) owing to variations in wind speeds throughout the year. This case study assumes that at every node in every time period, the output of every generator can drop to half the maximum output, that is $v_{ikt}^{L}=0.5v_{ikt}^{U}, \forall i\in S, k\in K, t\in\{1,\ldots,T\}$. This is a conservative approach (for instance, photovoltaic plants only lose half their output levels during the winter season) that accounts for all types of losses including seasonal and day-to-day variations, efficiency losses, downtimes due to maintenance, and complete breakdowns. To moderate the level of conservatism, a value of $\Gamma_{t}=1.3, \forall t\in\{1,\ldots,T\}$ is chosen.

The values of Γ_t have important implications in our model. For two vectors $\boldsymbol{\Gamma}^{*}$ and $\boldsymbol{\Gamma}^{**}$, where $\Gamma_{t}^{**}>\Gamma_{t}^{*}, \forall t\in\{1,\ldots,T\}$, $\mathcal{U}_{t}\left(\boldsymbol{v_{t}^{L}},\boldsymbol{v_{t}^{U}},\Gamma_{t}^{**}\right)\subset\mathcal{U}_{t}\left(\boldsymbol{v_{t}^{L}},\boldsymbol{v_{t}^{U}},\Gamma_{t}^{*}\right), \forall t\in\{1,\ldots,T\}$. Because of the immunization condition, first-stage decisions must be feasible over all uncertainty realizations. It is therefore clear that the feasible region of first-stage decision variables for the model with $\boldsymbol{\Gamma}^{*}\subseteq$ the feasible region of first-stage decision variables for the model with $\boldsymbol{\Gamma}^{**}$. Furthermore, for a first-stage solution that is feasible for both cases, the second-stage optimal value (the second stage being a maximization problem over a cartesian product of uncertainty sets) with $\boldsymbol{\Gamma}^{**}$ is less than or equal to that with $\boldsymbol{\Gamma}^{*}$. Therefore, the optimal value of the robust optimization model with $\boldsymbol{\Gamma}^{**}$ is less than or equal to that with $\boldsymbol{\Gamma}^{*}$. The usefulness of this conclusion will become clear in the next paragraph.

The AARC, though finite, is a big model and has long solution times. Using the same computer system as for the deterministic model and with the imposition of a 10-hour limit on solution times (this time limit is chosen for convenience and consistency since the optimal solution could not be found even after 48 h due to slow bound progressions. In addition, for most runs, the 10-hour optimal values are very close to the 48-hour ones.), the objective value obtained is $\$9.04\times 10^{8}$ and the optimality gap is 6.38%. Table 5 shows how the objective value and optimality gap obtained with CPLEX change when ρ, the value of Γ_t in every time period, changes. Values are shown to 5 decimal places to make small differences observable. Because of the knowledge that the optimal value of the robust optimization model with $\boldsymbol{\Gamma}^{**}$ is less than or equal to that with $\boldsymbol{\Gamma}^{*}$ when $\Gamma_{t}^{**}>\Gamma_{t}^{*}, \forall t\in\{1,\ldots,T\}$, it is possible to improve the CPLEX objective value. It is implicit that the proposed bound improvement is achieved heuristically here. Because of the slow bound improvements, we cannot verify the optimality of the improved bounds. That said, improving the bounds with such a method has other advantages. The improved objective value for run r is the minimum CPLEX objective value over all runs from 1 to r. By the same token, one can draw some rough satisficing conditions from the improved objective

Table 5
Sensitivity analysis on ρ.

Run	ρ	CPLEX objective value after 10 h ($\$ \times 10^8$)	Optimality gap	Improved objective value ($\$ \times 10^8$)
1	1.0	9.03719	0.00%	9.03719
2	1.1	9.03719	5.03%	9.03719
3	1.2	9.04126	6.10%	9.03719
4	1.3	9.03725	6.38%	9.03719
5	1.4	9.03725	7.01%	9.03719
6	1.5	9.03776	8.26%	9.03719
7	1.6	9.06800	9.77%	9.03719
8	1.7	9.28908	12.90%	9.03719
9	1.8	9.16634	18.50%	9.03719
10	1.9	7.85028	11.96%	7.85028
11	2.0	4.96806	0.00%	4.96806

value. If the improved objective value for run r is that of $r^* < r$, it implies that the solution of run r is also feasible for run r^* and therefore, that the generator and line setup solution of run r remains feasible even when the lower levels of conservatism in run r^* are imposed. Satisficing is a concept geared towards identifying solutions that remain satisfactory for a greater part of the feasibility space. Satisficing conditions are popular go-to alternatives when optimal solutions are hard to find, especially in the energy sector (Suranovic, 1993). They are rules of thumb that allow the planning of systems towards minimum aspirations or targets, accepting some sub-optimality in the process. While the improved bounds proposed may be sub-optimal, they may be preferable due to their validity over a larger number of uncertain events. This is reasonable, especially when planning robustness against black swans. The decision maker simply has no idea where the worst-case lies.

Table 6 shows the robust generator addition plan for the improved objective value of the AARC when $\Gamma_t = 1.3, \forall t \in \{1, \ldots, T\}$. Here (compared to the deterministic case), power generation amounts are omitted because these depend on uncertainty realization. In terms of transmission expansion, a new line is added from node 7 to 8 in the first time period and no lines are expanded.

4.3. Comparison of AARC and deterministic solution plans

The main reason for using robust optimization for power grid expansion planning is that probability distributions for generator supply capacities are difficult to estimate. While it is possible to find fitting distributions from load distribution curves when demand uncertainty is considered, the same cannot be said of supply uncertainties of variable generation resources, especially when considering multiple generator types and circumstances where generators are disrupted or become unavailable for other reasons. Even under regular working conditions, for losses in generator supply capacities due to inefficiencies, variations in environmental conditions, and equipment depreciation, it is quite impossible to find fitting probability distributions. To compare the performances of the deterministic model and its AARC, fix the first-stage variables to the respective best obtained solutions. For the deterministic model, the best first-stage solution is the one that gives the optimal value and for the AARC, it is the solution that gives the improved objective value. Scenarios of generator outputs are then randomly sampled. Bearing in mind the difficulty in finding fitting probability distributions, a uniform distribution in the range $\left[v_{ikt}^L, v_{ikt}^U\right]$ is used to sample every v_{ikt}. The performances of the generator and line setup solutions of the deterministic model and its AARC are then compared using 3 metrics: the number of infeasible instances among the scenarios, the average optimal value, and the standard deviation of the objective value.

For a set of 1000 scenarios, the deterministic model is always infeasible and the AARC gives an average optimal value of $\$8.87 \times 10^8$, an optimal value standard deviation of $\$1.16 \times 10^6$, and no infeasible instances (obvious since the generator and line setup solution of the improved objective value when $\Gamma_t = 1.3, \forall t \in \{1, \ldots, T\}$ is the optimal generator and line setup solution when all $\Gamma_t = 1.0, \forall t \in \{1, \ldots, T\}$). The solution of the deterministic model is therefore worse than the solution of its AARC in all instances. An infeasible instance, or an infeasible scenario, occurs when the generator supply capacity realization in that scenario makes it impossible to satisfy at least one constraint of the model. To show comparisons where the deterministic model actually has a reasonable number of feasible instances, a sample every v_{ikt} in the range $\left[0.94v_{ikt}^U, v_{ikt}^U\right]$ is needed (even with a lower bound of $0.93v_{ikt}^U$, more than half – 577 – of the 1000 scenario yield infeasible instances). Metric comparisons for this situation is shown in Table 7.

As seen from the table, the deterministic model far outperforms the AARC in terms of average objective value when generator output losses are very small. As expected though, the AARC offers better hedging against infeasibilities, with zero infeasible instances as compared to 351 for the deterministic solution.

Table 6
Robust generator addition plan.

	Supply nodes					
Period	Node 1	Node 2	Node 5	Node 8	Node 11	Node 13
1	– Set up a wind farm.	– No action.	– Set up a conventional combustion turbine.	– Set up a wind farm.	– No action	– No action.
2	– No action.	– No action.	– No action.	– No action.	– Set up a wind farm.	– No action.
3	– Set up a wind farm.	– No action.	– No action.	– No action.	– No action.	– No action.

Table 7
Metric comparisons between the deterministic model and its AARC for uniform sampling of every v_{ikt} in the range $\left[0.94v_{ikt}^U, v_{ikt}^U\right]$.

	Deterministic	AARC
Average objective value ($\$ \times 10^8$)	4.97	8.87
Objective value standard deviation ($\$ \times 10^6$)	0.11	1.16
Number of infeasible instances	351	0

4.4. Comparison of AARC and stochastic solution plans

While the deterministic model is useful to introduce the power grid capacity expansion planning methodology, it is, sensibly, not generally used for planning under uncertainty. Most of the literature on power grid expansion under uncertainty use stochastic programming approaches. This section shows how AARC solutions behave as compared to those of a stochastic programming alternative. The literature uses stochastic programming models with expected value objectives for power grid expansion planning. A direct comparison between these types of models with the AARC would be misleading. The AARC is clearly a planning methodology suited for risk averse decision makers, whereas the expected value objective is risk neutral. To allow risk aversion variations in stochastic programming, a conditional value-at-risk objective is used (see Rockafellar and Uryasev, 2000), or α-CVar for short. α-CVar is defined as the conditional expectation of a random variable given that its value is greater than or equal to α-Var, where α-Var is a lower α-percentile of the random variable. It is a popular method of incorporating risk aversion in stochastic programming because of its superior mathematical properties (convexity, monotonicity, positive homogeneity, and translational equivariance) to other methods. The α-CVar objective has been used in a wide array of applications, such as portfolio optimization (Krokhmal et al., 2002), energy trading (Dahlgren et al., 2003), and operational planning for large-scale industrial batch plants (Verderame and Floudas, 2009). The stochastic programming model for our power grid expansion problem with α-CVar objective, N scenarios, and all random variables sampled from uniform distributions is

$$\min\ \eta + \frac{1}{N(1-\alpha)} \sum_{\omega=1}^{N} \delta_\omega$$

$$\text{s.t.}\quad \delta_\omega \geq \sum_{t=1}^{T} \frac{1}{(1+r)^t} \left(\sum_{i\in S}\sum_{k\in K} c_{ikt}^G x_{ikt}^G + \sum_{(i,j)\in N^L} c_{ijt}^L y_{ijt}^L + \sum_{(i,j)\in V} c_{ijt}^E y_{ijt}^E + \sum_{i\in S}\sum_{k\in K} c_{ikt}^P g_{ikt\omega} \right) - \eta \quad \forall \omega \in \{1,\ldots,N\}$$

$$\text{s.t.}\quad s_{ij}(\theta_{it\omega} - \theta_{jt\omega}) = p_{ijt\omega} \quad \forall (i,j) \in V, t \in \{1,\ldots,T\}, \omega \in \{1,\ldots,N\}$$

$$\sum_{(i,j)\in\sigma_i^-} p_{ijt\omega} - \sum_{(j,i)\in\sigma_i^+} p_{jit\omega} = \sum_{k\in K} g_{ikt\omega} \quad \forall i \in S, t \in \{1,\ldots,T\}, \omega \in \{1,\ldots,N\}$$

$$\sum_{(j,i)\in\sigma_i^+} p_{jit\omega} - \sum_{(i,j)\in\sigma_i^-} p_{ijt\omega} = d_{it} \quad \forall i \in D, t \in \{1,\ldots,T\}, \omega \in \{1,\ldots,N\}$$

$$\sum_{(j,i)\in\sigma_i^+} p_{jit\omega} - \sum_{(i,j)\in\sigma_i^-} p_{ijt\omega} = 0 \quad \forall i \in I, t \in \{1,\ldots,T\}, \omega \in \{1,\ldots,N\}$$

$$g_{ikt\omega} \leq v_{ikt}\left(n_{ik} + \sum_{\tau=1}^{t} x_{ik\tau}^G \right) \quad \forall i \in S, k \in K, t \in \{1,\ldots,T\}, \omega \in \{1,\ldots,N\}$$

$$\sum_{i\in S}\sum_{k\in K} \gamma_k g_{ikt\omega} \leq (1-\beta)^t \sum_{i\in S}\sum_{k\in K} \gamma_k g_{ik0} \quad \forall t \in \{1,\ldots,T\}, \omega \in \{1,\ldots,N\}$$

$$|p_{ijt\omega}| \leq f_{ij}^L \sum_{\tau=1}^{t} y_{ij\tau}^L + f_{ij}^E \sum_{\tau=1}^{t} y_{ij\tau}^E \quad \forall (i,j) \in N^L, t \in \{1,\ldots,T\}, \omega \in \{1,\ldots,N\}$$

$$|p_{ijt\omega}| \leq f_{ij}^L + f_{ij}^E \sum_{\tau=1}^{t} y_{ij\tau}^E \quad \forall (i,j) \in E^L, t \in \{1,\ldots,T\}, \omega \in \{1,\ldots,N\}$$

$$y_{ijt}^E \leq y_{ijt}^L \quad \forall (i,j) \in N^L, t \in \{1,\ldots,T\}$$

$$\sum_{t=1}^{T} y_{ijt}^L \leq 1 \quad \forall (i,j) \in N^L$$

$$\sum_{t=1}^{T} y_{ijt}^E \leq 1 \quad \forall (i,j) \in V$$

$$\text{All } y_{ijt}^L, y_{ijt}^E \in \{0,1\}, \quad \text{all } x_{ikt}^G \in \mathbb{Z}_+, \text{all remaining variables} \in \mathbb{R}_+.$$

Uniform distributions are used on all uncertain parameters to generate scenarios. Following the sample average approximation method (see Kleywegt et al., 2002), a popular stochastic programming solution method applied in a wide variety of areas such as vehicle routing (Verweij et al., 2003) and supply chain design (Schütz et al., 2009), the model is solved for 200 scenarios for 1 h to obtain an approximate

Table 8
Metric comparisons between AARC and stochastic with different risk aversions and uniform random sampling of all v_{ikt} in $\left[0.5v_{ikt}^{U}, \zeta v_{ikt}^{U}\right]$.

			AARC	α 0.0	0.2	0.4	0.6	0.8
ζ	1.0	Average objective value ($\$ \times 10^8$)	8.89	7.63	7.64	7.66	7.57	7.64
		Objective value standard deviation ($\$ \times 10^6$)	1.16	2.19	2.21	1.86	1.92	2.07
		Number of infeasible instances	0	11	10	227	76	10
	0.9	Average objective value ($\$ \times 10^8$)	8.89	7.64	7.65	7.68	7.58	7.65
		Objective value standard deviation ($\$ \times 10^6$)	1.06	2.12	2.12	1.63	1.57	2.11
		Number of infeasible instances	0	12	10	417	109	19
	0.8	Average objective value ($\$ \times 10^8$)	8.91	7.66	7.67	7.69	7.59	7.67
		Objective value standard deviation ($\$ \times 10^6$)	0.89	1.80	1.80	1.18	1.23	1.89
		Number of infeasible instances	0	47	40	792	260	31
	0.7	Average objective value ($\$ \times 10^8$)	8.92	7.68	7.69	7.71	7.60	7.69
		Objective value standard deviation ($\$ \times 10^6$)	0.69	1.22	1.22	0.00	0.95	1.20
		Number of infeasible instances	0	204	151	999	622	169
	0.6	Average objective value ($\$ \times 10^8$)	8.94	7.70	7.71	N/A	N/A	7.71
		Objective value standard deviation ($\$ \times 10^6$)	0.99	0.40	0.39	N/A	N/A	0.38
		Number of infeasible instances	0	973	952	1000	1000	965
	0.5	Average objective value ($\$ \times 10^8$)	9.04	N/A	N/A	N/A	N/A	N/A
		Objective value standard deviation ($\$ \times 10^6$)	N/A	N/A	N/A	N/A	N/A	N/A
		Number of infeasible instances	0	1000	1000	1000	1000	1000

solution which is then used to fix first-stage variables. The model with fixed first-stage variables is then run over 1000 scenarios to calculate the average optimal solution, the standard deviation of the optimal solution, and the number of infeasible instances. The results for different values of α, representing different risk aversion levels, are reported in Table 8. Comparisons are also made between the models under different severity of generator output losses.

The stochastic programming model gives a better average optimal value over all runs. However, the AARC is significantly more effective at hedging against infeasibilities. The AARC shows no infeasible instances among the different simulation runs. The difference in the number of infeasible instances between the AARC and the stochastic programming model is even more pronounced when generator output losses are greater. For example, when all generators lose from 30% to 50% of their outputs, the stochastic programming models under all our chosen risk aversion levels (α) yield at least 151 infeasible instances. In this work, the aim is to build resilience in power grids. While it is important to have strong performances, one has to accept losses in the average performance to build power grids that remain feasible in extreme or unpredictable events. Black swans, as such events are called, are usually, and ineffectively, planned for with the benefit of hindsight. Here a proactive decision process is offered that reinforces the power grid and keeps them functional under such events.

4.5. Comparison of AARC and a fully-immunized stochastic programming model

The reason for the numerous infeasibilities in the above subsection is that the stochastic programming model is only feasible for the scenarios that it is immunized against. The first-stage solution that is feasible for the 200 scenarios initially generated in the sample average approximation method may not be feasible (is not feasible, in fact, for many instances) for some of the 1000 scenarios used to test the second-stage decisions. A fully-immunized stochastic programming model would be one where the first-stage decisions are feasible for all 1000 scenarios. This quality is ensured by making the stochastic programming model feasible for a scenario where all $v_{ikt} = v_{ikt}^{L}$, that is, by ensuring that at least one of the 200 initial scenarios of the sample average approximation is one where all $v_{ikt} = v_{ikt}^{L}$. This levels the performance of the stochastic programming model with the AARC in terms of the number of infeasible instances (0 infeasible instances for both) and therefore, allows a fair comparison between the objective value averages and standard deviations. Similar to the previous subsection, the model is solved for 200 scenarios, where one scenario has all $v_{ikt} = v_{ikt}^{L}$, for 1 h to obtain an approximate solution which is then used to fix first-stage variables. The model with fixed first-stage variables is then run over 1000 scenarios to calculate the average optimal solution and the standard deviation of the optimal solution. Obviously, there are no infeasible instances in the fully-immunized stochastic programming model. Results for different risk aversions are shown in Table 9.

The AARC generally outperforms the stochastic programming model. The average objective value is better and so is the standard deviation of the objective value (except for the case where $\alpha = 0.6$). One would expect the stochastic programming model, at least for $\alpha = 0.0$, to outperform the AARC in terms of average optimal value, since its objective itself is to minimize the average optimal value. The reason for the counter-intuitive results is that the sample average approximation method does not yield the optimal value of the stochastic programming model. Because the model is effectively a multi-stage model, the number of scenarios needed to fix the first-stage decisions cannot be large and cause intractability. Even with 200 scenarios to generate first-stage decisions, the optimal solution is not found within 1 h (the best result among all the α variations is an optimality gap of around 9%). Fig. 2 is a visual comparison between the AARC and the stochastic programming model with $\alpha = 0.0$ (SP_0) for each of the 1000 scenarios. It shows the clear superiority of the AARC results in almost all instances.

Table 9
Metric comparisons between AARC and fully-immunized stochastic with different risk aversions.

	AARC	α 0.0	0.2	0.4	0.6	0.8
Average objective value ($\$ \times 10^8$)	8.89	8.92	9.01	8.93	10.8	10.1
Objective value standard deviation ($\$ \times 10^6$)	1.16	1.41	1.42	1.73	1.11	1.83

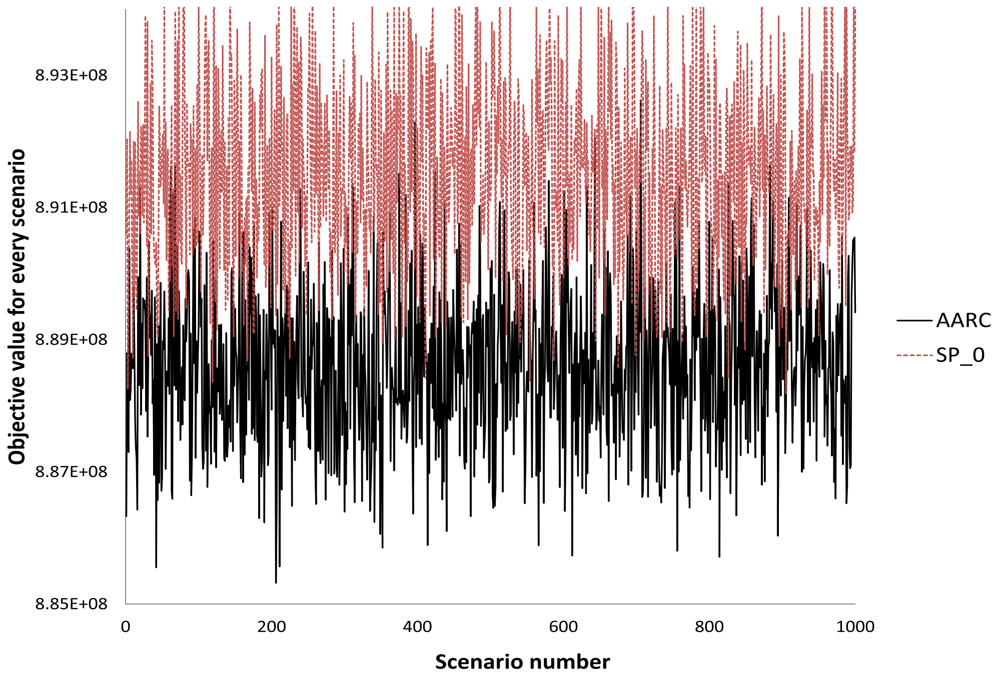

Fig. 2. Comparison between the results of the AARC and the stochastic programming model with $\alpha = 0.0$ (SP_0) for every scenario.

A note of caution must be articulated here on comparisons between robust optimization and stochastic programming. These two methods propound different optimization philosophies. Robust optimization is generally geared towards better extreme- or worst-case performances whereas stochastic programming aims to find better average performances. Since the focus of this paper is on planning under black swans, that is, extreme unpredictable events, the setting in which the robust optimization and stochastic programming models are compared is an extreme one, in the sense that the worst-case scenario in the stochastic programming model is deemed as likely to happen as other scenarios. This has an adverse effect on the performance of the stochastic programming model. In less extreme situations, the stochastic programming is likely to perform better than in the one used in this paper.

5. Conclusion

This paper constructs a multi-period two-stage adaptive robust optimization model for long term power grid capacity expansion in an environmentally conscious way. The aim is to build expansion plans that are robust to uncertainties in generator outputs due to natural variations and generator disruptions. The first stage is used to plan robust generator and line addition/expansion. The second stage performs power generation and dispatch flexible to uncertainty realizations, in a way that minimizes the worst case generation cost. The uncertainty characterizations are in the form of deterministic uncertainty sets, with parameters that permits control over the level of conservatism.

The resulting two-stage model is semi-infinite and intractable. Because the model does not have relatively complete recourse, available methods are inadequate to solve it exactly. An affinely adjustable robust counterpart (AARC) is therefore devised, where recourse decisions are approximated as affine functions of uncertainty realizations. The AARC is a finite single-stage mixed integer programming model which can be solved with generic software. The benefits of using robust optimization are demonstrated in a case study of the IEEE 30-bus system. The AARC results are compared with those of its deterministic equivalent, showing great improvement in the number of infeasible instances from uniformly distributed generator output realizations. Only in cases where output losses are minimal does the deterministic equivalent perform better than the AARC. In addition, the AARC results are compared with those of a stochastic programming model with a conditional value-at-risk objective to allow for different risk aversion levels, showing that while the stochastic programming model gives better optimal values on average over uncertainty realizations, it yields significantly more infeasible instances, especially when generator losses are high. This shows the greater resilience injected in power grids by the AARC. The AARC is also compared with a fully-immunized stochastic programming model, that is, one that has no infeasible instances, showing that the AARC clearly outperforms the fully-immunized stochastic programming model when a sample average approximation is used to solve the latter.

This work only considers operational flexibility. Generation and power dispatch plans change with uncertainty realizations. Building on this platform, the future work would be to add strategic flexibility to the expansion plan. This entails the addition of options on generator and line additions, with an even more complex mixed integer second stage. Furthermore, the model currently devises plans for phased expansion over the long term. However, there are no insights into how generator addition should be undertaken within a time period. In addition, the model assumes that the growth in load requirements is known with certainty. If this new uncertainty dimension is added, strategic flexibility could yield stronger gains since it would wait on uncertainty to happen before setting up generators and lines, which have expensive setup costs.

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